Lesson 8-6: Perimeter & Area of Similar Figures

It stands to reason doesn't it?

Perimeter and area are both derived from the lengths of the sides of a figure. If the sides are proportional for two figures, what conjecture would you make about how the perimeter of one figure relates to the other? What about how the area of one figure relates to the other?

If you haven't done so yet, take a few minutes and work through the investigation exercise. It will help you answer this question.

Perimeters and areas of similar figures

When you worked through the investigation, you discovered the ratio of the perimeters is the same as the figures' similarity ratio. You also discovered the ratio of the areas is the same as the square of the similarity ratio. From this we derive a theorem…

Theorem 8-6 Perimeters and areas of similar figures

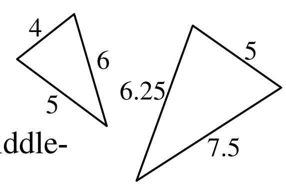
If two figures are similar and have a similarity ratio of $\frac{a}{b}$, then

1. similarity ratio = the ratio of the figures' perimeters ($\frac{a}{b} = \frac{P_1}{P_2}$)
2. (similarity ratio)^2 = the ratio of the figures' areas ($\frac{a^2}{b^2} = \frac{A_1}{A_2}$)

With the 2nd, think about it this way…the area is units^2…the area is already squared.

Examples

1. These triangles are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.



   The first step is to determine corresponding sides. Match up sides by relative length: shortest-shortest, longest-longest, middle-middle: $\frac{5}{4} = \frac{7.5}{6} = \frac{6.25}{5}$

   Next determine the similarity ratio. Pick the easiest ratio… 5 : 4 or $\frac{5}{4}$.

   The ratio of the perimeters is the same as the similarity ratio: $\frac{5}{4}$.

   The ratio of the areas is the square of the similarity ratio: $\frac{5^2}{4^2} = \frac{25}{16}$

2. The ratio of the lengths of the corresponding sides of two regular octagons is $\frac{8}{3}$. The area of the larger octagon is 320 ft^2. Find the area of the smaller octagon.

   The similarity ratio is $\frac{8}{3}$. The ratio of the area is then $\frac{64}{9}$. Therefore:

   $\frac{320}{A} = \frac{64}{9}$; $64 \bullet A = 320 \bullet 9$; $A = \frac{2880}{64} = 45\ ft^2$

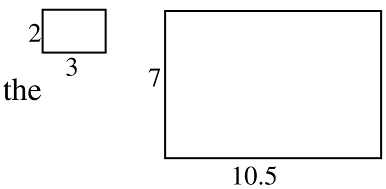
Lesson 8-6: Perimeter & Area of Similar Figures

3. Benita plants the same crop in two rectangular fields, each with side lengths in a ratio of 2 : 3. Each dimension of the larger field is 3½ times the dimension of the smaller field. Seeding the smaller field costs \$8. How much money does seeding the larger field cost?

   The phrase "*each with side lengths in a ratio of 2 : 3*" is important. This means, for each field, the width to length ratio is 2 : 3.

   Next we are told the ratio of corresponding sides of the larger field to the smaller is 3.5 : 1. This is the similarity ratio for the fields.

   Great! Let's draw a picture of this…



   The cost of seeding a field is based on the area of the field. In other words, the cost is proportional to the area. So we need to use a ratio of the areas which is the square of the similarity ratio:

   $$\frac{cost\,large\ field}{cost\,small\ field} = \frac{area\,large\ field}{area\,small\ field} = \frac{3.5^2}{1^2}$$

   $$\frac{c}{8} = \frac{3.5^2}{1^2};\ c = 8 \bullet 3.5^2 = 8 \bullet 3.5 \bullet 3.5 = \$98$$

4. The areas of two similar pentagons are 32 in^2 and 72 in^2. What is their similarity ratio? What is the ratio of their perimeters?

   The area ratio is $\frac{32}{72} = \frac{4}{9}$. This ratio is equal to the square of the similarity ratio:

   Similarity ratio: $\frac{4}{9} = \frac{a^2}{b^2};\ \frac{a}{b} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

   Ratio of perimeters: $\frac{2}{3}$

5. The similarity ratio of two similar triangles is 5 : 3. The perimeter of the smaller triangle is 36 *cm*, and its area is 18 cm^2. Find the perimeter and area of the larger triangle.

   Ratio of perimeters = similarity ratio: $\frac{P_{large}}{P_{small}} = \frac{5}{3};\ \frac{P_{large}}{36} = \frac{5}{3};\ P_{large} = \frac{36 \bullet 5}{3} = 60\,cm$

   Ratio of areas = (similarity ratio)^2: $\frac{A_{large}}{A_{small}} = \frac{5^2}{3^2} = \frac{25}{9};\ \frac{A_{large}}{18} = \frac{25}{9}; A_{large} = \frac{18 \bullet 25}{9} = 50\,cm^2$

Homework Assignment

Pg 456 - #1-22, 24, 25-32, 35-37, 40-44